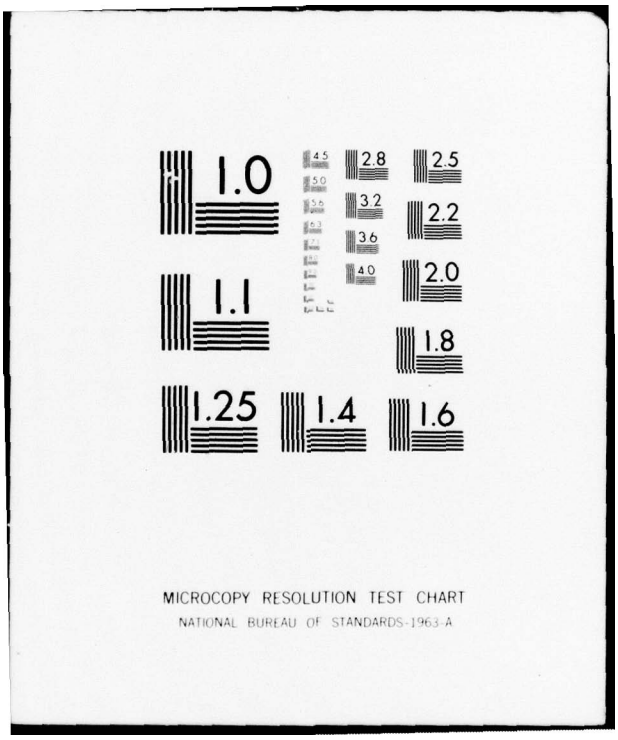
AD-A038 641 MOORE SCHOOL OF ELECTRICAL ENGINEERING PHILADELPHIA P--ETC F/G 9/4
ROBUST ARRAY DETECTORS.(U)
1977 S A KASSAM

UNCLASSIFIED AF-AFOSR-3154-77
AFOSR-TR-77-0550 NL

| OF |
AD
A038641



END
DATE
FILMED
5-77


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6
6.3
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AFOSR - TR - 77 - 0550

ROBUST ARRAY DETECTORS

Saleem A. Kassam
University of Pennsylvania
The Moore School of Electrical Engineering
Department of Systems Engineering
Philadelphia, PA. 19104

AD A038641

ABSTRACT

Array detectors based on correlator-type structures are considered for robust detection of weak signals with noise represented by a generalized Tukey-Huber contamination model. The performance of a different type of quadratic limiter-sum detector for robust detection, obtained through a consideration of the locally optimum detector for robust detection, is also discussed.

I. INTRODUCTION

Robust statistical procedures for signal detection problems exhibit performances which are fairly stable over classes of statistical characterizations of the signals and noise. They are therefore useful in implementing detectors which have to operate under some uncertainty as to the exact nature of the signal and noise processes. The known signal in additive noise problem has been considered by several investigators (e.g. [1]-[3]). Techniques for robust detection of random signals from multiple inputs have been less thoroughly investigated, although nonparametric detectors have been given more attention [4], [5]. Robust correlator-type detectors have been considered in [6] and [7]; in [6], two-input correlators were studied and some limiter structures were investigated for performance, and in [7] a general robust detection problem was formulated.

In this paper we show that if attention is restricted to correlator-type structures and detector performance is characterized in terms of an asymptotic measure, the efficacy [8], robust detectors for random signals in additive noise can be explicitly defined. We will use the Tukey-Huber contamination noise model to define the class of noise statistics; extensions of this model allow us to look at the case of noise processes which are correlated between the several input channels. In addition, we also briefly consider for robust detection a modified quadratic-sum detector which performs better for small arrays.

DDC FILE COPY

for public release;
tion unlimited.

II. ROBUST CORRELATORS

A. Preliminaries

For detecting a random signal common to an array of m channels, we assume that a total of n vector samples $\underline{X}_1$, $\underline{X}_2,\ldots,\underline{X}_n$ are taken, where $\underline{X}_i = (X_{i1}, X_{i2},\ldots,X_{im})$ is the vector of outputs at each of the m channels at the i-th time instant; thus X_{ij} is the output of the j-th channel at the i-th sampling time. If θS_i represents a random signal common to the array at the i-th sampling time, we can describe the m-variate probability density function $f_{\underline{X}|\theta}$ of $\underline{X}_i$ by

$$f_{\underline{X}|\theta}(\underline{x}) = \int f_{\underline{X}}(\underline{x} - \theta\underline{s})dF_S(s). \qquad (1)$$

In (1), F_S is the distribution function of S_i and is assumed, without loss of generality, to have unit variance; the vector $\underline{s}$ is an m-component vector with each component equal to s; and $f_{\underline{X}}$ is the m-variate density function of noise across the m channels. With this notation, $f_{\underline{X}|0}=f_{\underline{X}}$. We have assumed that the statistics of the signal and noise are stationary in time, and we will also assume that under the null hypothesis H_0: $\theta=0$, the time samples $\underline{X}_i$ are independent. This independence of the noise samples is necessary for the rest of the development.

For the two-input case (m=2) the general correlator detector [6] is based on the test statistic

$$S_2 = \sum_{i=1}^{n} w(X_{i1})w(X_{i2}), \qquad (2)$$

where w is a suitable function. Extending this for the m-input case, we now consider a correlator-array test statistic S_m defined by

$$S_m = \sum_{i=1}^{n}\sum_{j=1}^{m-1}\sum_{k=j+1}^{m} w(X_{ij})w(X_{ik}) . \qquad (3)$$

We will consider later the restriction as well as justification for the assumption

Presented at the Conference on Information Sciences and Systems, The Johns Hopkins University, March 1977; to be published in the Proceedings of the Conference.

DDC
APR 28 1977

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

(19) REPORT NUMBER: AFOSR - TR - 77 - 0550	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): ROBUST ARRAY DETECTORS.	5. TYPE OF REPORT & PERIOD COVERED: Interim rept.
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Saleem A. Kassam	8. CONTRACT OR GRANT NUMBER(s): AFOSR 77-3154
9. PERFORMING ORGANIZATION NAME AND ADDRESS: University of Pennsylvania, Moore Sch. of E E, Dept of Systems Engineering, Philadelphia, PA 19104	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F 2304/A5
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Office of Scientific Research/NM, Bolling AFB, Washington, DC 20332	12. REPORT DATE: 1977
	13. NUMBER OF PAGES: 6
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): 12 6p.	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

15 AF-AFOSR-3154-77

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NSF-ENG-76-10444

18. SUPPLEMENTARY NOTES

16 2304 19 A5

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Array detectors based on correlator-type structures are considered for robust detection of weak signals with noise represented by a generalized Tukey-Huber contamination model. The performance of a different type of quadratic limiter-sum detector for robust detection, obtained through a consideration of the locally optimum detector for robust detection, is also discussed.

DD FORM 1473 EDITION OF 1 NOV 65 IS OBSOLETE

409814

of this structure for a multi-input test statistic.

Under rather general conditions, the ARE of two detectors can be expressed as a ratio of their efficacies [8]. Thus our measure of performance will be the efficacy of a correlator array. The efficacy E_m of the test statistic S_m is given by

$$E_m = \lim_{n\to\infty} \frac{\left[\frac{\partial}{\partial\theta} E_\theta\{S_m\}\,|_{\theta=0}\right]^2}{n \operatorname{var}_0\{S_m\}} \quad . \qquad (4)$$

If we assume now that $f_{\underline{X}}$, the m-variate density of the array noise vector, is a product of m independent and identical symmetric, univariate densities f, i.e.

$$f_{\underline{X}}(\underline{x}) = \prod_{i=1}^{m} f(x_i) \ , \qquad (5)$$

we have, assuming that $f'(x)$ is sufficiently smooth and, (without loss of generality), that $h(X_{ij})$ has zero mean under H_0, the result that

$$E_m = \frac{m(m-1)}{2} \frac{[\int w(x) f'(x)dx]^4}{[\int w^2(x) f(x)dx]^2} \qquad (6)$$

On the other hand, if w is sufficiently smooth and bounded, we obtain

$$E_m = \frac{m(m-1)}{2} \frac{[\int w'(x) f(x)dx]^4}{[\int w^2(x) f(x)dx]^2} \ . \qquad (7)$$

Equation (7) is the result which we will use later.

From (6), Schwarz's inequality immediately leads to the conclusion that $w(x) = c\, f'(x)/f(x)$, where c is any constant, gives the optimum correlator-array when f is known. We now consider robust correlators for situations where f, and more generally $f_{\underline{X}}$, is not completely specified.

The contaminated noise model initially used by Tukey and Huber for robustness studies [9], [10] (and later used to formulate signal detection problems) provides one way of characterizing a class of densities for the noise. The univariate models which have been used so far model a density f as a mixture of a known nominal component g and an arbitrary contamination h;

$$f(x) = (1-\varepsilon)g(x) + \varepsilon\, h(x) \ , \qquad (8)$$

where ε is an assumed degree of contamination parameter between 0 and 1. The robust detection problem is then to find a "worst-case" h, say h_R, and corresponding density $f=f_R$, for which the optimum detector D_R is derived. D_R is then the most robust detector if for any f, the performance of D_R (here measured by its efficacy) is lower bounded by its (optimum) performance for f_R.

We now consider the model (8) and some extensions for our present application.

B. Robust Correlators for Contaminated Noise

The first noise model for the multi-input detection problem we are considering is obtained by combining the assumption of (5) with the model (8); we will call the result Model 1. The resulting noise class contains m-variate densities for the array noise vector of the form

$$f_{\underline{X}}(\underline{x}) = \prod_{i=1}^{m} [(1-\varepsilon)g(x_i) + \varepsilon h(x_i)]. \qquad (9)$$

We will assume, for Model 1 and for the other models we will consider, that the nominal density g is symmetric, $-\log g(x)$ is convex (as for the Gaussian density) and $g'(x)$ is absolutely continuous (see [2]). We also assume for Model 1 that h is bounded and symmetric.

Since (9) is still a product of univariate densities, the efficacy $E_{m,1}$ of S_m for $f_{\underline{X}}$ a density given by Model 1 is

$$E_{m,1} = \frac{m(m-1)}{2} \frac{\left[(1-\varepsilon)\int w'(x)g(x)dx + \varepsilon\int w'(x)h(x)dx\right]^4}{\left[(1-\varepsilon)\int w^2(x)g(x)dx + \varepsilon\int w^2(x)h(x)dx\right]^2} \qquad (10)$$

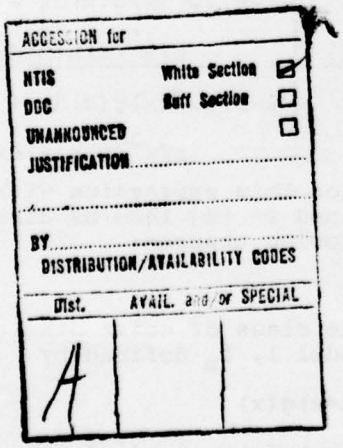
Comparison of this expression with results obtained in [2] lead us directly to the following theorem:

Theorem 1

For the class of noise densities given by Model 1, f_R defined by

$$f_R(x) = \begin{cases} (1-\varepsilon)g(x) & |x|<a \\ (1-\varepsilon)g(a)\exp[-b(|x|-a)] & |x|\geq a \end{cases} \qquad (11)$$

where a, b satisfy $\int_{-a}^{a} g(x)dx + 2g(a)/b = (1-\varepsilon)^{-1}$ and $-g'(a)/g(a) = b$, and the


ACCESSION for
NTIS White Section
DDC Buff Section
UNANNOUNCED
JUSTIFICATION
BY
DISTRIBUTION/AVAILABILITY CODES
Dist. AVAIL. and/or SPECIAL
A

corresponding nonlinearity

$$w_R(x) = -f_R'(x)/f_R(x) \qquad (12)$$

constitute the least favorable density and the maxi-min correlator-array nonlinearity, respectively, w_R giving the most robust performance in terms of efficacy.

The proof is quite simple; from (11), we see that h_R is zero in $(-a,a)$, so that with w_R replacing w in (10), the denominator is maximized by h_R. Also, for w_R in the numerator, h_R minimizes the value of the numerator, $w_R'(x)$ being zero for $|x|>a$ and non-negative for $|x|<a$. By (12), we conclude that w_R results in the optimum correlator for f_R, and therefore the theorem is proved. It should be noted that in [7] the nature of such a saddle point solution has been discussed for a general convex family of noise densities (here generated by Model 1).

It is interesting that the robust nonlinearity w_R is identical to the robust nonlinearity for the known signal in additive noise detector problem considered in [2]; it is true, however, that for the known signal problem the false-alarm probability was kept bounded, whereas robustness of performance in terms of efficacy does not necessarily imply a controlled false-alarm probability for a fixed-threshold test.

We now consider a different extension of the univariate contamination model of (8), which does not give channel-to-channel independence for the noise. With g and h density functions satisfying the same conditions as in Model 1, we define Model 2 to give m-variate noise density functions

$$f_{\underline{X}}(\underline{x}) = (1-\varepsilon)\prod_{i=1}^{m} g(x_i) + \varepsilon \prod_{i=1}^{m} h(x_i). \qquad (13)$$

This results in marginal bivariate densities for uncorrelated random variables; however, it is clear that these are not independent. It is easily seen that the maximal correlation coefficient [11], which is a much more suitable measure of dependence between random variables, may be strictly bounded away from zero. Thus for Model 2, ε is the degree of contamination and dependence.

For Model 2, it follows easily that the efficacy $E_{m,2}$ of the correlator-array statistic (3) is given by

$$E_{m-2} = \frac{m(m-1)}{2} \frac{\left[(1-\varepsilon)\left(\int w^1(x)g(x)dx\right)^2 + \varepsilon\left(\int w^1(x)h(x)dx\right)^2\right]^2}{(1-\varepsilon)\left[\int w^2(x)g(x)dx\right]^2 + \varepsilon\left[\int w^2(x)h(x)dx\right]^2} \qquad (14)$$

A comparison of (14) with (10), and an examination of the proof of Theorem 1 allows us to state the following theorem:

Theorem 2

For the class of noise densities given by Model 2, w_R defined in (12) results in a correlator-array statistic which is robust in the sense that its efficacy $E_{m,2}$ is always larger than or equal to the efficacy obtained with that $h=h_R$ which results in the f_R of (11).

The proof of this theorem is quite simple. Note that we cannot conclude that the density $f_{\underline{X},R}(\underline{x})$ given by

$$f_{\underline{X},R}(\underline{x}) = (1-\varepsilon)\prod_{i=1}^{m} g(x_i) + \varepsilon \prod_{i=1}^{m} h_R(x_i)$$

and w_R constitute a saddle-point solution for the robust detection problem, as in Theorem 1. This is because it has not been shown that w_R is the optimum correlator-array nonlinearity for the multivariate noise density of (14). The result is nevertheless very useful because it gives more significance to the robust nonlinearity w_R of Theorem 1; with w_R, the resulting performance remains lower bounded by a computable efficacy for all densities in Model 2 as well. Some numerical values of the performances for noise densities generated by Models 1 and 2 are shown in Table I-III.

The numerical values shown were computed for the particular case where g is a zero mean, unit variance Gaussian density. Table I shows the lower bounds on the efficacies (normalized by dividing by $\frac{m(m-1)}{2}$) when w_R is the correlator array nonlinearity, as a function of the contamination parameter ε. The two lower bounds (for densities in the two models) are very close, and they decrease with increasing ε, as is to be expected. Of course, the lower bounds are achieved for a rather special kind of contamina-

tion, so that, for example, if the contamination density is also unimodal, the resulting performance will usually be considerably better. It is seen that the limiter-type nonlinearity can be expected to give reasonably good performance for all densities generated by Models 1 and 2, for small contamination levels ε.

In Table II, normalized efficacies are shown for a specific type of contamination in Models 1 and 2; the density function h was taken to be Gaussian with variance σ^2, with $\sigma^2 \geq 1$, to observe the effect of broader tailed contaminations. In Table III, for comparison, the same contaminations are applied to the nominal density for the correlator-array which is optimal for the nominal density, i.e. with $w(x)=x$. It is clear that the performance of the optimum array deteriorates rapidly, especially for Model 2, as the noise density starts getting non-Gaussian.

We can extend Model 2 for a contaminated multivariate nominal density by relaxing the requirement that the multivariate contaminating density $h(\underline{x})$ *is a product* $\prod_{i=1}^{m} h(x_i)$ *of univariate* densities. Thus we consider Model 3 for contaminated multivariate densities $f_{\underline{X}}(\underline{x})$, which are defined for this model by

$$f_{\underline{X}}(\underline{x}) = (1-\varepsilon) \prod_{i=1}^{m} f(x_i) + \varepsilon h_{\underline{X}}(\underline{x}) \qquad (15)$$

We now want to see if the robust correlator for Models 1 and 2 also has a lower-bounded performance for noise densities in this larger class of densities than that generated by Model 2. Clearly, some restrictions will have to be imposed on $h_{\underline{X}}$.

Suppose we assume that $f_{\underline{X}}$ and hence $h_{\underline{X}}$ describes spatially homogeneous and symmetrically distributed noise, by which we mean that any k-variate marginal density of the channel noise vector is the same k-variate function, and the value of the noise density is invariant under changes of sign for any k of its m arguments, for k=1,2,...,m. Under the homogeneity and symmetry assumptions, we can show that the robustness of the correlator-array nonlinearity w_R for Model 2 also holds for the larger class of noise densities of Model 3. In spite of the conditions of symmetry and homogeneity, Model 3 generates a strictly larger class of densities than does Model 2; for example one can include $h_{\underline{X}}$ in Model 3 which are themselves mixtures of contaminations of the form $\prod_{i=1}^{m} h(x_i)$.

Finally, it should be noted that it is also possible to show that a lower bound on performance exists for the correlator-array using w_R even without the spatial homogeneity assumption. This is a direct extension of the above results. For example, in a geometrically linear array, it is more reasonable to assume that the bivariate inter-channel noise densities could depend on the spacing between the channel receivers. Such considerations will modify the expression for the derivative of the mean and variance of the test statistic, the $\frac{m(m-1)}{2}$ terms now not being all identical. It will still be true, however, that the performance, in terms of efficacy, of the correlator-array using w_R is lower bounded as before.

III. ROBUST ARRAY FROM LOCALLY-OPTIMUM ARRAYS

All the previous results were obtained for the case where the signal detection structure was constrained to be of the correlator type. One of the consequences of this is that no results were obtained for the one-channel random signal problem. Theorem 1 tells us that w_R is the most robust nonlinearity subject to this correlator structure constraint. We will now consider briefly a robust detector structure motiviated by the form of the locally-optimum array detector.

Applying the generalized Neyman-Pearson lemma [8], we find that the locally-optimum detector for a random signal in additive noise in m channels is based on the test statistic T given by

$$T = \sum_{i=1}^{n} \left[\tfrac{1}{2} \sum_{j=1}^{m} \frac{f''(X_{ij})}{f(X_{ij})} + \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} \frac{f'(X_{ij})}{f(X_{ij})} \frac{f'(X_{ik})}{f(X_{ik})} \right], \qquad (16)$$

assuming f'' exists almost everywhere. It is clear from (16) that the optimum array detector for weak signals is not a correlator in general; in fact, T

reduces to a correlator-array only for the double-exponential density, giving the polarity-coincidence array [4]. If m is fairly large, the m second derivative terms in (16) may be ignored, being a minor contribution in comparison with the m(m-1) correlation terms. Thus for large m, a correlator-structure assumption is quite reasonable. The correlator-array detector has generally received considerable attention because of its relative ease of implementation, although it is known that in the two-input case [12], for example, its ARE relative to the locally-optimum detector is quite poor. This is correspondingly true for robust detection based on the correlator-structure as opposed to the locally-optimum structure.

Consider the case of Gaussian noise with unit variance; then the statistic T becomes, after dropping constant terms,

$$T_g = \sum_{i=1}^{n} \left(\sum_{j=1}^{m} X_{ij} \right)^2$$

$$= \sum_{i=1}^{n} \left(\sum_{j=1}^{m} X_{ij}^2 + 2 \sum_{j=1}^{m-1} \sum_{k=j+1}^{m} X_{ij} X_{ik} \right). \quad (17)$$

This can also be written as

$$T_g = \sum_{i=1}^{n} \sum_{j=1}^{m} \left[\frac{f'(X_{ij})}{f(X_{ij})} \right]^2 , \quad (18)$$

where f here is the unit-variance Gaussian density. The structure of (18) leads us to investigate the robustness properties of the test statistic T_R defined by

$$T_R = \sum_{i=1}^{n} \sum_{j=1}^{m} [w_R(X_{ij})]^2 . \quad (19)$$

As a matter of fact, one may consider any reasonable limiter-type nonlinearity instead of w_R; this is motivated by the well known robustness properties of such nonlinearities. Note that T_R gives the same kind of performance as observed in the previous section when m is large. For the one-input case, T_R reduces to the test statistic investigated in [13]. Numerical studies of the performance of T_R confirm the expected. The performance of the detector T_R for contaminated noise densities (in the numerical study, the contaminations were Gaussian with variance >1) was considerably better than the robust correlator array for small m; for m>6, the performances were not very different. The T_R of (19) was motivated purely by the previous results, and it is not the most robust statistic in that a saddlepoint for performance is not obtained with T_R.

IV. SUMMARY

Robust correlator structures were considered for random signal detection in additive noise. The criterion of performance used was the efficacy of a detector test statistic, a measure of the weak-signal detection performance of the statistic. For a direct multivariate extension of the Tukey-Huber contaminated noise model, it was shown that the max-min robust nonlinearity is the same as the robust nonlinearity for known signals in contaminated additive noise. The contamination noise model was then generalized to allow non-independent multivariate contaminations. It was shown that the same robust nonlinearity provides stable performance for the resulting class of noise densities. Finally, we considered the performance of an array detector for robust detection for small arrays. The quadratic limiter-sum detector for Gaussian noise was found to have better stable performance for contaminated noise in small arrays, and its performance was close to that of the robust correlators in moderate-sized arrays.

ACKNOWLEDGEMENTS

This research is supported by the National Science Foundation under Grant ENG76-10444 and by the Air Force Office of Scientific Research, Air Force Systems Command, USAF, under Grant AFOSR-77-3154.

REFERENCES

1. R. D. Martin and S. C. Schwartz, "Robust detection of a known signal in nearly Gaussian noise," IEEE Trans. Inform. Theory, Vol. IT-17, pp. 50-56, Jan. 1971.

2. S. A. Kassam and J. B. Thomas, "Asymptotically robust detection of a known signal in contaminated non-Gaussian noise," IEEE Trans. Inform. Theory, Vol. IT-22, pp. 22-26, Jan. 1976.

3. A. H. El-Sawy and V. D. VandeLinde, "Robust detection of known signals," TR 76-5, Elect. Eng. Dept. Johns Hopkins University.

4. M. Kanefsky, "Detection of weak signals with polarity coincidence arrays," IEEE Trans. Inform. Theory,

Vol. IT-12, pp. 260-268, April 1966.

5. S. A. Kassam and J. B. Thomas, "Array detectors for random signals in noise," IEEE Trans. Sonics & Ultrasonics, Vol. SU-23, pp. 107-112, March 1976.
6. S. S. Wolff and J. L. Gastwirth, "Robust two-input correlators," J. Acoust. Soc. Amer., Vol. 41, pp. 1212-1219, May 1967.
7. S. S. Wolff and J. M. Sullivan, "Robust many-input signal detectors," in Signal Processing, J.W.R. Griffiths, et al., Eds., London; Academic Press, 1973.
8. J. Capon, "On the asymptotic efficiency of locally optimum detectors," IRE Trans. Inform. Th., Vol. IT-7, pp. 67-71, April 1961.
9. J. W. Tukey and T. E. Harris, "Sampling from contaminated distributions," 3," Memo Rept. 31, Statist. Res. Group, Princeton Univ., 1949.
10. P. J. Huber, "Robust estimation of a location parameter," Ann. Math. Statist., Vol. 35, pp. 73-101, March 1964.
11. A. N. Kolmogorov and Y. A. Rozanov, "On strong mixing conditions for stationary Gaussian processes," Th. Prob. Appl., Vol. 5, pp. 204-209, 1960.
12. S. S. Wolff, J. B. Thomas and T. R. Williams, "The polarity coincidence correlator: a nonparametric detection device," IRE Trans. Inform. Th., Vol. IT-8, pp. 5-9, Jan. 1962.
13. R. D. Martin and C. P. McGath," Robust detection of stochastic signals," IEEE Trans. Inform. Th., Vol. IT-20, pp. 537-541, July 1974.

TABLE I

Lower Bounds on Normalized Efficacies for Correlator Based on w_R; Unit-Variance Gaussian Nominal Density

ε	0.02	0.04	0.06	0.08	0.10
Lower Bound Model 1	0.802	0.683	0.590	0.514	0.450
Lower Bound Model 2	0.755	0.666	0.602	0.550	0.507

TABLE II

Normalized Efficacies for Correlator Based on w_R; Gaussian Contaminations, Variance σ^2, with ε=0.05

σ^2	1	4	9	16
Efficacy, Model 1	0.913	0.787	0.736	0.710
Efficacy, Model 2	0.913	0.781	0.728	0.702

TABLE III

Normalized Efficacies for Optimum Correlator Based on w(x)=x; Gaussian Contaminations, Variance σ^2, ε=0.05

σ^2	1	4	9	16
Efficacy, Model 1	1.000	0.756	0.510	0.326
Efficacy, Model 2	1.000	0.571	0.200	0.073